

# Finite

## U-46 Curriculum Scope and Sequence

Reporting Strand	Instructional Focus	Standards	Semester
Matrices	Represent linear equations in matrices.	A.REI.8, A.REI.9	1
	Perform operations on matrices and use matrices in applications.	N.VM.6, N.VM.7,N.VM.8, N.VM.9, N.VM.10, N.VM.11, N.VM.12	
Linear Programming	Use geometric linear programming to solve problems.	A.REI.6, A.REI.12, A.CED.1, A.CED.3	1
	Use algebraic linear programming to solve problems.	A.REI.8, A.REI.9	
Applied Matrix Theory	Evaluate and analyze Markov Chains	A.REI.8	1
	Use Game Theory to solve problems.	S.MD.5, S.MD.6, S.MD.7	
Financial Math	Analyze and apply different types of interest and rate	A.SSE.1, A.CED.2, A.CED.4, F.BF.5, F.LE.3, F.IF.6	2
Probability	Calculate expected values and use them to solve problems	S.MD.1, S.MD.2, S.MD.3, S.MD.4	2
Statistics	Analyze and use data to solve problems	AP Stats Prep	2

## Matrices

### Instructional Focus: Representing linear equations

CCSS	4 – Mastery	3 – Proficient	2 - Basic	1 – Below Basic	0 – No Evidence
<b>Representing and finding inverses of matrices</b> (A.REI.8, A.REI.9)	Can extend thinking beyond the standard, including tasks that may involve one of the following: <ul style="list-style-type: none"> <li>• Designing</li> <li>• Connecting</li> <li>• Synthesizing</li> <li>• Applying</li> <li>• Justifying</li> <li>• Critiquing</li> <li>• Analyzing</li> <li>• Creating</li> <li>• Proving</li> </ul>	Represent a system of equations using matrices when variables are <b><u>on both sides of an equation, or have missing variables.</u></b>  Find the inverse of a matrix and use it solve systems of linear equations with dimensions of <ul style="list-style-type: none"> <li>• 2x2 <b><u>without</u></b> technology</li> <li>• 3x3 with technology</li> </ul>	Represent a system of equations using matrices when all variables are <b><u>on one side of each equation.</u></b>  Find the inverse of a matrix <b><u>and use it solve systems of linear equations with dimensions of</u></b> <ul style="list-style-type: none"> <li>• 2x2 <b><u>with</u></b> technology</li> <li>• 3x3 with technology</li> </ul>	<b><u>Identify</u></b> a system of equations in a matrix. Find the inverse of a matrix	Little evidence of reasoning or application to solve the problem  Does not meet the criteria in a level 1

A.REI.9 Find the inverse of a matrix if it exists and use it to solve systems of linear equations.

A.REI.8 Represent a system of linear equations as a single matrix equation in a vector variable.

## Matrices

Instructional Focus: Perform operations on matrices and use matrices in applications.

CCSS	4 – Mastery	3 – Proficient	2 - Basic	1 – Below Basic	0 – No Evidence
<b>Matrix operations and applications</b> (N.VM.6, N.VM.7, N.VM.8, N.VM.11)	Can extend thinking beyond the standard, including tasks that may involve one of the following: <ul style="list-style-type: none"> <li>• Designing</li> <li>• Connecting</li> <li>• Synthesizing</li> <li>• Applying</li> <li>• Justifying</li> <li>• Critiquing</li> <li>• Analyzing</li> <li>• Creating</li> <li>• Proving</li> </ul>	Extract a matrix or matrices from a situation (i.e. word problem) <b>and use the matrix or matrices to solve problems.</b>  Given matrices, do <b>all</b> of the following with and without solving technology: <ul style="list-style-type: none"> <li>• Multiply by scalars</li> <li>• Add matrices</li> <li>• Subtract matrices</li> <li>• Multiply matrices</li> <li>• Multiply by a vector</li> </ul>	Extract a matrix or matrices from a situation (i.e. word problem)  Given matrices, do <b>all</b> of the following with solving technology: <ul style="list-style-type: none"> <li>• Multiply by scalars</li> <li>• Add matrices</li> <li>• Subtract matrices</li> <li>• Multiply matrices</li> <li>• Multiply by a vector</li> </ul>	<b>Identify</b> the corresponding matrix from a situation.  Given matrices, do <b>three</b> of the following with solving technology : <ul style="list-style-type: none"> <li>• Multiply by scalars</li> <li>• Add matrices</li> <li>• Subtract matrices</li> <li>• Multiply matrices</li> <li>• Multiply by a vector</li> </ul>	Little evidence of reasoning or application to solve the problem  Does not meet the criteria in a level 1
<b>Explaining properties of matrices</b> (N.VM.9, N.VM.10)		Can explain <b>all</b> of the following: <ul style="list-style-type: none"> <li>• Lack of Commutative property of Matrix Multiplication</li> <li>• Associative property of Matrix Multiplication</li> <li>• Distributive property of Matrix Multiplication</li> <li>• Zero Matrix</li> <li>• Identity Matrix</li> </ul>	Can explain <b>four</b> of the following: <ul style="list-style-type: none"> <li>• Lack of Commutative property of Matrix Multiplication</li> <li>• Associative property of Matrix Multiplication</li> <li>• Distributive property of Matrix Multiplication</li> <li>• Zero Matrix</li> <li>• Identity Matrix</li> </ul>	Can explain <b>three</b> of the following: <ul style="list-style-type: none"> <li>• Lack of Commutative property of Matrix Multiplication</li> <li>• Associative property of Matrix Multiplication</li> <li>• Distributive property of Matrix Multiplication</li> <li>• Zero Matrix</li> <li>• Identity Matrix</li> </ul>	
<b>Finding and using determinants and absolute values</b> (N.VM.12)		Find the area by using the determinant and absolute value of a $2 \times 2$ matrix as a transformation on the plane.	Find determinant and absolute value of a $2 \times 2$ matrix as a transformation on the plane.	Find determinant and absolute value of a $2 \times 2$ matrix	

N.VM.6 Use matrices to represent and manipulate data.

N.VM.7 Multiply matrices by scalars to produce new matrices.

N.VM.8 Add, subtract, and multiply matrices of appropriate dimensions.

N.VM.11 Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.

N.VM.9 Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.

N.VM.10 Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers.

N.VM.12 Work with  $2 \times 2$  matrices as a transformations of the plane, and interpret the absolute value of the determinant in terms of area.

## Linear Programing

### Instructional Focus: Geometric Linear Programing

CCSS	4 – Mastery	3 – Proficient	2 - Basic	1 – Below Basic	0 – No Evidence
<p><b>Solving systems of equations and inequalities</b> (A.REI.6)</p> <p><b>Graphing systems of equations and inequalities</b> (A.REI.6, A.REI.12)</p> <p><b>Creating equations</b> (A.CED.1*)</p> <p><b>Representing constraints and interpreting solutions</b> (A.CED.3*)</p>	<p>Can extend thinking beyond the standard, including tasks that may involve one of the following:</p> <ul style="list-style-type: none"> <li>• Designing</li> <li>• Connecting</li> <li>• Synthesizing</li> <li>• Applying</li> <li>• Justifying</li> <li>• Critiquing</li> <li>• Analyzing</li> <li>• Creating</li> <li>• Proving</li> </ul>	<p>Graph the feasible region based on constraints</p> <p>Find each vertex of the feasible region by solving a system of equations</p> <p>State the feasible region as being bounded or unbounded</p> <p>Represent constraints with equations, inequalities and in a system of equations and/or inequalities in contextual situations</p> <p>Create the objective function with two or more variables from context <u>and use it in a linear programming problem to find the optimal solution</u></p> <p>Interpret test points <u>as viable or nonviable in context of the situation</u></p>	<p><u>Graph</u> the feasible region based on constraints.</p> <p><u>Find</u> each vertex of the feasible region <u>by solving a system of equations</u></p> <p>State the feasible region as being bounded or unbounded</p> <p>Represent constraints with equations, inequalities and in a system of equations and/or inequalities in contextual situations</p> <p><u>Create</u> the objective function with two or more variables from context</p> <p>Interpret test points <u>in context of the situation</u></p>	<p><u>Identify</u> the feasible region given a graph</p> <p><u>Identify</u> each vertex of the feasible region given a graph</p> <p>State the feasible region as being bounded or unbounded</p> <p><u>Identify</u> the objective function with two or more variables for a given context</p>	<p>Little evidence of reasoning or application to solve the problem</p> <p>Does not meet the criteria in a level 1</p>

**A.REI.6** **Solve** systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

**A.CED.1\*** **Create** equations and inequalities in one variable and **use** them to solve problems

**A.CED.3\*** **Represent** constraints by equations or inequalities, and by systems of equations and/or inequalities; and interpret solutions as viable or nonviable options in a modeling context.

## Linear Programing

### Instructional Focus: Algebraic Linear Programing

CCSS	4 – Mastery	3 – Proficient	2 - Basic	1 – Below Basic	0 – No Evidence
<b>Solving linear programming problems using matrices</b> (A.REI.8, A.REI.9)	Can extend thinking beyond the standard, including tasks that may involve one of the following: <ul style="list-style-type: none"> <li>• Designing</li> <li>• Connecting</li> <li>• Synthesizing</li> <li>• Applying</li> <li>• Justifying</li> <li>• Critiquing</li> <li>• Analyzing</li> <li>• Creating</li> <li>• Proving</li> </ul>	Represent a system of given constraints using a matrix <ul style="list-style-type: none"> <li>• Identify an optimized problem</li> <li>• Identify the pivot</li> <li>• Find the solution (<u>more than 1 pivot required</u>)</li> <li>• Interpret the tableau in context of the situation</li> </ul> <u>Create a system of optimized constraints from a context</u>	Represent a system of given constraints using a <u>2x2 or 3x3</u> matrix <ul style="list-style-type: none"> <li>• Identify an optimized problem</li> <li>• Identify the pivot</li> <li>• Find the solution using the simplex method (1 pivot required)</li> <li>• Interpret the tableau <u>in context</u> of the situation</li> </ul>	Represent a system of given constraints using a <u>2x2</u> matrix <ul style="list-style-type: none"> <li>• Identify an optimized problem</li> <li>• Identify the pivot</li> <li>• Find solution using the simplex method (1 pivot required)</li> <li>• Interpret <u>the parts</u> of the tableau</li> </ul>	Little evidence of reasoning or application to solve the problem  Does not meet the criteria in a level 1

A.REI.8 Represent a system of linear equations as a single matrix equation in a vector variable

A.REI.9 Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension  $3 \times 3$  or greater).

## Applied Matrix Theory

### Instructional Focus: Markov Chains

CCSS	4 – Mastery	3 – Proficient	2 - Basic	1 – Below Basic	0 – No Evidence
<b>Creating and interpreting Markov chains</b> (A.REI.8)	Can extend thinking beyond the standard, including tasks that may involve one of the following: <ul style="list-style-type: none"> <li>• Designing</li> <li>• Connecting</li> <li>• Synthesizing</li> <li>• Applying</li> <li>• Justifying</li> <li>• Critiquing</li> <li>• Analyzing</li> <li>• Creating</li> <li>• Proving</li> </ul>	Create a transition matrix and distribution vector <u>from context</u>  Find and <u>interpret</u> the steady state distribution, distribution after n transitions (regular or absorbing), <u>and</u> probability of being absorbed	Create a transition matrix and distribution vector <u>from context</u>  Find the steady state distribution <u>or</u> the distribution after n transitions	Create a transition matrix <u>from a diagram</u>  Classify given matrices by type	Little evidence of reasoning or application to solve the problem  Does not meet the criteria in a level 1

A.REI.8 Represent a system of linear equations as a single matrix equation in a vector variable.

## Applied Matrix Theory

### Instructional Focus: Game Theory

CCSS	4 – Mastery	3 – Proficient	2 - Basic	1 – Below Basic	0 – No Evidence
<b>Creating and analyzing matrices in Game Theory</b> (S.MD.5, S.MD.6, S.MD.7)	Can extend thinking beyond the standard, including tasks that may involve one of the following: <ul style="list-style-type: none"> <li>• Designing</li> <li>• Connecting</li> <li>• Synthesizing</li> <li>• Applying</li> <li>• Justifying</li> <li>• Critiquing</li> <li>• Analyzing</li> <li>• Creating</li> <li>• Proving</li> </ul>	For zero sum games including <b>at least two</b> options <b>without a saddle point</b> <ul style="list-style-type: none"> <li>• Create a payoff matrix</li> <li>• Find the mixed strategy (probability distributions) for each player</li> <li>• Find the expected value of the game</li> </ul>	For zero sum games including two options with <b>more than one</b> saddle point <ul style="list-style-type: none"> <li>• Create a payoff matrix</li> <li>• Find the <b>mixed strategy</b> (probability distributions) for each player</li> <li>• Find the expected value of the game</li> </ul>	For zero sum games including two options with <b>a</b> saddle point <ul style="list-style-type: none"> <li>• Create a payoff matrix</li> <li>• Find the strategy (probability distributions) for each player</li> <li>• Find the expected value of the game</li> </ul>	Little evidence of reasoning or application to solve the problem  Does not meet the criteria in a level 1

S.MD.5 Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.

S.MD.6 Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).

S.MD.7 Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

## Financial Math

## Instructional Focus: Analyze and apply different types of interest and rate

CCSS	4 – Mastery	3 – Proficient	2 - Basic	1 – Below Basic	0 – No Evidence
<b>Interpret Expressions</b> (A.SSE.1)	Can extend thinking beyond the standard, including tasks that may involve one of the following: <ul style="list-style-type: none"> <li>• Designing</li> <li>• Connecting</li> <li>• Synthesizing</li> <li>• Applying</li> <li>• Justifying</li> <li>• Critiquing</li> <li>• Analyzing</li> <li>• Creating</li> <li>• Proving</li> </ul>	Interpret individual parts of expressions (such as variables, coefficients, factors, etc.) and explain their meaning in terms of the context in <b>all of the following</b> : <ul style="list-style-type: none"> <li>• Simple Interest</li> <li>• Compound Interest</li> <li>• Annuities</li> </ul> Group parts of an expression and interpret their meaning in terms of the context in <b>all of the following</b> : <ul style="list-style-type: none"> <li>• Simple Interest</li> <li>• Compound Interest</li> <li>• Annuities</li> </ul>	Interpret individual parts of expressions (such as variables, coefficients, factors, etc.) <b>and explain their meaning in terms of the context in two of the following</b> : <ul style="list-style-type: none"> <li>• Simple Interest</li> <li>• Compound Interest</li> <li>• Annuities</li> </ul> Group parts of an expression and interpret their meaning <b>in terms of the context in two of the following</b> : <ul style="list-style-type: none"> <li>• Simple Interest</li> <li>• Compound Interest</li> <li>• Annuities</li> </ul>	Interpret individual parts of expressions (such as variables, coefficients, factors, etc.) in all of the following: <ul style="list-style-type: none"> <li>• Simple Interest</li> <li>• Compound Interest</li> <li>• Annuities</li> </ul> Group parts of an expression and interpret their meaning in all of the following: <ul style="list-style-type: none"> <li>• Simple Interest</li> <li>• Compound Interest</li> <li>• Annuities</li> </ul>	Little evidence of reasoning or application to solve the problem  Does not meet the criteria in a level 1
<b>Create and solve equations</b> (A.CED.2 A.CED.4)		Create and solve equations to represent relationships in contextual situations, including <b>all</b> the following situations: <ul style="list-style-type: none"> <li>• Simple Interest</li> <li>• Compound Interest</li> <li>• Annuities</li> <li>• Amortization</li> </ul>	<b>Create and solve equations</b> to represent relationships in contextual situations, including <b>two</b> the following situations: <ul style="list-style-type: none"> <li>• Simple Interest</li> <li>• Compound Interest</li> <li>• Annuities</li> <li>• Amortization</li> </ul>	<b>Create and solve equations</b> to represent relationships in contextual situations, in <b>one</b> of the following situations: <ul style="list-style-type: none"> <li>• Simple Interest</li> <li>• Compound Interest</li> <li>• Annuities</li> <li>• Amortization</li> </ul>	
<b>Exponential and Logarithmic inverses</b> (F.BF.5)		Recognize that exponential and logarithmic functions are inverses of each other and use these functions to <b>solve real-world problems</b> .	Recognize that exponential and logarithmic functions are inverses of each other and use these functions <b>to solve logarithmic and exponential equations</b> .	Recognize that exponential and logarithmic functions are inverses of each other and <b>convert from one form into the other</b> .	
<b>Compare Rate of Change</b> (F.LE.3, F.IF.6)		Calculate and compare the rate of change and value of function presented in symbolic and table form in context of a situation <b>and use it to make a decision</b> <ul style="list-style-type: none"> <li>• Stated rate</li> <li>• Effective rate</li> </ul>	Calculate and compare the rate of change and value of function presented in symbolic <b>and</b> table form <b>in context of a situation</b> <ul style="list-style-type: none"> <li>• Stated rate</li> <li>• Effective rate</li> </ul>	Calculate the rate of change and value of a function presented in symbolic <b>or</b> table form <ul style="list-style-type: none"> <li>• Stated rate</li> <li>• Effective rate</li> </ul>	

A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law  $V = IR$  to highlight resistance  $R$ .

A.SSE.1 Interpret expressions that represent a quantity in terms of its context. ★  
 a. Interpret parts of an expression, such as terms, factors, and coefficients.  
 b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret  $P(1+r)^n$  as the product of  $P$  and a factor not depending on  $P$ .

F.BF.5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. \*(Modeling Standard)

F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★



## Probability

Instructional Focus: Calculate expected values and use them to solve problems

CCSS	4 – Mastery	3 – Proficient	2 - Basic	1 – Below Basic	0 – No Evidence
<b>Representing probability distributions</b> (S.MD.1)	<p>Can extend thinking beyond the standard, including tasks that may involve one of the following:</p> <ul style="list-style-type: none"> <li>• Designing</li> <li>• Connecting</li> <li>• Synthesizing</li> <li>• Applying</li> <li>• Justifying</li> <li>• Critiquing</li> <li>• Analyzing</li> <li>• Creating</li> <li>• Proving</li> </ul>	<p><u>Define a random variable for a quantity of interest</u></p> <p>Assign a numerical value to each event in a sample space</p> <p>Graph the corresponding probability distribution using the same graphical displays as for data distributions.</p>	<p><u>Assign a numerical value to each event in a sample space</u></p> <p>Graph the corresponding probability distribution <u>using the same graphical displays as for data distributions.</u></p>	Graph a given probability distribution	<p>Little evidence of reasoning or application to solve the problem</p> <p>Does not meet the criteria in a level 1</p>
<b>Calculating and interpreting expected values</b> (S.MD.2)		<p><u>Calculate and interpret</u> the expected value of a random variable and use the information to make a decision</p>	Calculate the expected value of a random variable <u>and use the information to make a decision</u>	Calculate the expected value of a random variable	
<b>Developing probability distributions and finding expected values</b> (S.MD.3, S.MD.4)		<p>Develop a probability distribution for a random variable for a sample space of</p> <ul style="list-style-type: none"> <li>• theoretical probabilities</li> <li>• experimental probabilities</li> </ul> <p><u>and find the expected value</u></p>	<p><u>Develop</u> a probability distribution for a random variable for a sample space of</p> <ul style="list-style-type: none"> <li>• theoretical probabilities</li> <li>• experimental probabilities</li> </ul>	<p><u>Calculate</u> probabilities for a sample space of</p> <ul style="list-style-type: none"> <li>• theoretical probabilities</li> <li>• experimental probabilities</li> </ul>	

S.MD.1 Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.

S.MD.2 Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.

S.MD.3 Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value

S.MD.4 Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value.

## Statistics

### Instructional Focus: Analyze and use data to solve problems

CCSS	4 – Mastery	3 – Proficient	2 - Basic	1 – Below Basic	0 – No Evidence
<p><b>Calculating and interpreting standard deviations</b></p> <p><b>Determining the probability of normal distributions</b></p>	<p>Can extend thinking beyond the standard, including tasks that may involve one of the following:</p> <ul style="list-style-type: none"> <li>• Designing</li> <li>• Connecting</li> <li>• Synthesizing</li> <li>• Applying</li> <li>• Justifying</li> <li>• Critiquing</li> <li>• Analyzing</li> <li>• Creating</li> <li>• Proving</li> </ul>	<p>For random variables and binomial random variables, calculate <u>and interpret</u> the standard deviation</p> <p>Determine <u>binomial probability</u> by using normal approximation</p>	<p>For random variables <u>and</u> binomial random variables, calculate the standard deviation</p> <p>Determine the <u>probability of nonstandard normal distributions</u> by <u>calculating a z-score</u></p>	<p>For random variables <u>or</u> binomial random variables, calculate the standard deviation</p> <p>Determine the <u>probability of standard normal distributions</u>, <u>given a z-score</u></p>	<p>Little evidence of reasoning or application to solve the problem</p> <p>Does not meet the criteria in a level 1</p>